Лекция 3. Определение состава равновесной плазмы. Уравнения ионизационного равновесия.

Цель. Описать методы определения состава плазмы

$$F = F(T, V, N_i, N_e^*, N_0)$$

N_i, N_e^*, N_0, N_e - Число ионов, свободных электронов, атомов и полное число электронов

Для плазмы водорода

$$N_e = N_e^* + N_0, \quad N_n = N_i + N_0, \quad N^*_e = N_i$$

N_e, N_n константы

N_0, N_i, N_e^* переменные

$$\Rightarrow \delta N_0 = -\delta N_e^* = -\delta N_i$$

$$\frac{\partial F}{\partial N_i}\delta N_i + \frac{\partial F}{\partial N_e^*}\delta N_e^* + \frac{\partial F}{\partial N_0}\delta N_0 = 0$$

$$\left.\begin{aligned} &\frac{\partial F}{\partial N_i} + \frac{\partial F}{\partial N_e^*} = \frac{\partial F}{\partial N_0} \\ &\mu_k = \left(\frac{\partial F}{\partial N_k}\right)_{T,V} \end{aligned}\right\} \Rightarrow \quad \mu_i + \mu_e^* = \mu_0$$

For noninteracting particles chemical potential is:

$$\mu_k = \mu_k^0(T) + k_B T \ln n_k$$

$$\frac{n_0}{n_e^* n_i} = \exp\left[\frac{1}{k_B T}(\mu_i^{(0)} + \mu_e^{(0)} - \mu_0^{(0)})\right] = K(T)$$

$$\mu_k^{(0)} = -k_B T\left(\ln U_k(T,V)\frac{1}{\Lambda_k^3}\right)$$

$$U_k = \sum_s \exp\left[-\frac{E_s}{k_B T}\right] g_s \qquad \Lambda_k = h/\sqrt{2\pi m_k k_B T}$$

For free electron $U_e=2$

$$\frac{n_0}{n_e^* n_i} = \Lambda^3 \frac{U_0(T)}{2U_i(T)}$$

$$\Lambda = h/\sqrt{2\pi m_e k_B T}\sqrt{\frac{m_0}{m_i}} \approx h/\sqrt{2\pi m_e k_B T}$$

$$U_0(T) = 2U_i(T)\sigma(T),$$

$$\sigma(T) = \sum_{nl}(2l+1)\exp(-E_{nl}/k_B T)$$

$$\frac{n_0}{n_e^* n_i} = \Lambda^3\sigma(T) = K(T)$$ **Saha equation**

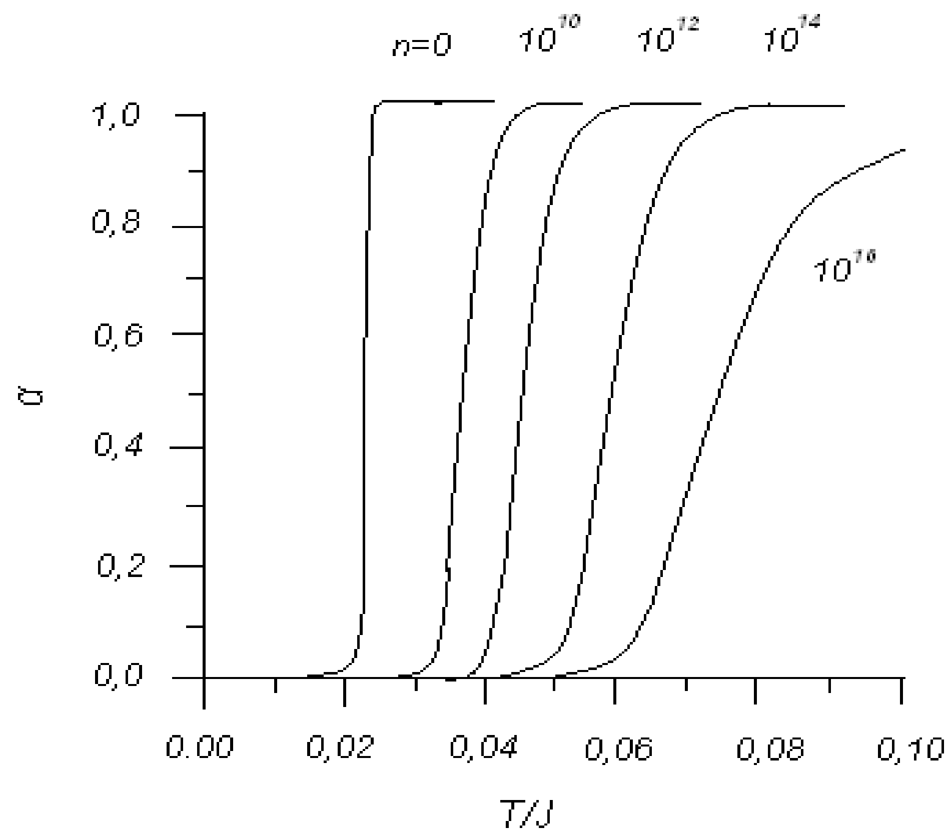
For hydrogen plasma $\sigma(T) = \sum_{n=1}^{\infty} \mathrm{n}^2 \exp(-E_n/k_B T)$

$$E_n = -\frac{I}{n^2}, \quad I = \frac{me^4}{2\hbar^2}, \quad \sigma(T) = \sum_{n=1}^{\infty} n^2 \exp(I/(k_B T n^2))$$

Problem of cut-off of quantum-mechanical statistic sum

Система ионизационный уравнений

$$\begin{cases} \dfrac{n_0}{n_e^* n_i} = \Lambda^3 \exp(I \;\; / k_B T) \\ n_i = n_e^* \\ n_0 + n_i = n_n \end{cases}$$



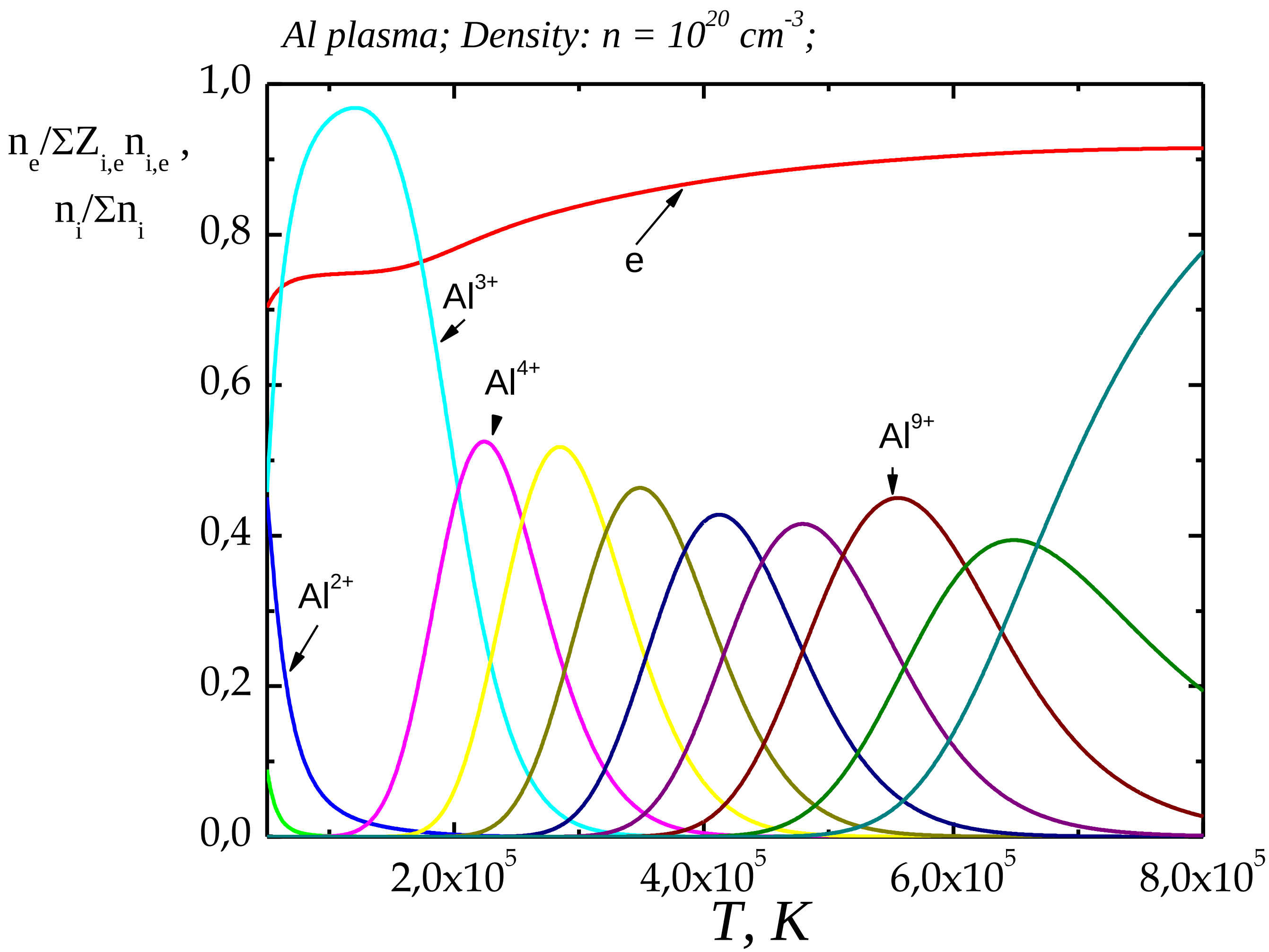
n=0
10^10
10^12
10^14
10^16
1,0
0,8
0,6
0,4
0,2
0,0
α
0.00
0,02
0,04
0,06
0,08
0,10
T/J

$$\mu_K = \mu_K^0(T) + k_B T \ln n_K + (\mu_K)^{\mathrm{int}}$$

$$(\mu_K)^{\mathrm{int}} = -\frac{Ze^2}{2r_D} = \Delta I$$

$$\frac{n_0}{n_e^* n_i} = K(T)\exp(\Delta I / k_B T) = K_{eff}$$

$$K_{eff} = \begin{cases} K(T)\exp(-\beta e^2/(2r_D)), & r_D \geq a_0, \\ 0, & r_D < a_{0.} \end{cases}$$



Al plasma; Density: n = 10^20 cm^-3;
$n_e/\Sigma Z_{i,e} n_{i,e}$,
$n_i/\Sigma n_i$
1,0
0,8
0,6
0,4
0,2
0,0
$2{,}0x10^5$
$4{,}0x10^5$
$6{,}0x10^5$
$8{,}0x10^5$
T, K
e
Al^{2+}
Al^{3+}
Al^{4+}
Al^{9+}